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A Construction of the Groups of Units of Some Number Fields from Certain Subgroups

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Introduction

All number fields we consider are in the complex number field. The symbol $\langle S\rangle$ denotes a multiplicative group generated by an element or a set S. For any complex number x, a j-th root of x, which is taken to be positive real if x is positive real, is denoted by $\sqrt[j]{x}$.

0.1. For a finite algebraic number field k, let E_k be the group of units of k and W_k be the torsion part of E_k. Then E_k is generated by W_k and by a set $\{\varepsilon_j \mid j=1, \cdots, r\}$ of fundamental units of k. The number r is called the *Dirichlet number* of k. In general, some geometrical calculation is necessary to obtain fundamental units of k (see [1] or Chap. 2, §5.3 of [2]). Those methods are very complicated when r is large. If k is a real abelian number field, there is an effective method, which requires no geometrical calculation, to obtain fundamental units of k (see [5]). Our main interest is, in case k is not galois or galois but not abelian over $\boldsymbol{Q}$, to construct $\{\varepsilon_j \mid j=1, \cdots, r\}$ from certain subgroups of E_k without any geometrical calculation. Let E_k' be the subgroup of E_k generated by W_k and the units of all proper subfields of k. If the index $(E_k: E_k')$ is finite, we may construct E_k from E_k'. Such a problem is treated in some cases when k is galois over $\boldsymbol{Q}$, see [6] and [7] for example. If $(E_k: E_k')$ is not finite, we consider the following subgroup H_k, the group of *relative units* of k, in addition to E_k':

$$H_k=\{\varepsilon \in E_k \mid N_{k/k_1}(\varepsilon) \in W_k \text{ for any proper subfield } k_1 \text{ of } k\}.$$

The object of the present article is to show a way how E_k is constructed from E_k' and H_k in some cases. Our main tool is Proposition 1 in §1, which can be applied to k of types as in 1.2. To explain our actual calculation, we take for k a subfield of a dihedral extension of

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