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Standard Subgroups of Type $G_2(3)$

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Introduction

A quasisimple subgroup L of a finite group G is said to be *standard* if $|C_G(L)|$ is even, $|C_G(L)\cap C_G(L)^g|$ is odd for each $g\in G-N_G(L)$, and $[L, L^g]\neq 1$ for each $g\in G$. Let $G_2(3^n)$ denote the Chevalley group of type (G_2) over the finite field $GF(3^n)$. The objective of this paper is to prove the following theorem.

THEOREM. *Let G be a finite group which possesses a standard subgroup L such that $L/Z(L)\cong G_2(3)$. Assume that $C_G(L)$ has a cyclic Sylow 2-subgroup and that $LO(G)\ntrianglelefteq G$. Then one of the following holds.*

(1) $E(G)\cong G_2(9)$.

(2) $E(G)/Z(E(G))\cong G_2(3)\times G_2(3)$.

(3) $N_G(L)/C_G(L)\cong \mathrm{Aut}\,(G_2(3))$ *and for an involution z of L, $C_G(z)$ has a quasisimple subgroup K which satisfies the following conditions:*

(i) $z\in K$, $O_2(K)$ *is cyclic of order* 4, *and* $K/O(K)\cong SU_4(3)$.

(ii) $[K, O(C_G(z))]=1$.

(iii) $K/\langle z\rangle$ *is a standard subgroup of* $C_G(z)/\langle z\rangle$ *and* $O_2(K)$ *is a Sylow 2-subgroup of* $C_G(K/\langle z\rangle)$.

We remark that Case (3) does not occur in any known examples of G. Thus it is anticipated that once the classification of finite groups with a standard subgroup of type $PSU_4(3)$ is established, Case (3) will be eliminated. This paper represents a contribution to the program of classifying all finite groups having a standard subgroup of known type.

As usual the method used in the proof is essentially a detailed analysis of 2-local subgroups of G depending heavily on the structure of 2-local subgroups of $G_2(3)$. In this context the group $G_2(3)$ seems to be "small". There are two reasons. First, $G_2(3)$ is of characteristic 2-type (a group X is said to be of *characteristic 2-type* provided $F^*(Y)=$

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