TOKYO J. MATH.
VOL. 5, NO. 1, 1982

A Local Isotopy Lemma

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(Communicated by K. Ogiue)

Introduction

First of all, let us recall Thom's first isotopy lemma: *Let $\pi: E\to B$ be a proper differentiable and stratified map. Then for each stratum Z of B, the restricted map $\pi: \pi^{-1}(Z)\to Z$ is a locally trivial fibration* (see R. Thom [5], J. Mather [3]).

This lemma is very powerful in the topological studies of analytic sets (see T. Fukuda [1]), of Landau singularities, of Feynman integrals (see F. Pham [4]) and so on. However we meet many situations where the mapping $\pi: E\to B$ is not proper, and we can not apply the lemma to the situations.

For example, consider the function $I(z)$ $(z=(z_1, \cdots, z_4)\in C^4)$ defined by the integral

$$I(z)=\int_0^{z_1}(z_2-2z_3\tau+z_4\tau^2)^{-1}d\tau \ .$$

It is the solution of the Cauchy problem:

$$\left\{\left(\frac{\partial}{\partial z_1}\right)^2+2z_3\left(\frac{\partial}{\partial z_1}\right)\left(\frac{\partial}{\partial z_2}\right)+z_4\left(\frac{\partial}{\partial z_1}\right)\left(\frac{\partial}{\partial z_3}\right)\right\}I(z)=0$$

with initial data

$$I(0, z_2, z_3, z_4)=0\ , \qquad \frac{\partial}{\partial z_1}I(0, z_2, z_3, z_4)=\frac{1}{z_2}\ .$$

Obviously $I(z)$ is holomorphic as long as the integral path with initial point $\tau=0$ and with terminal point $\tau=z_1$ can be continuously deformed, escaping the singularities of the integrand, $z_2-2z_3\tau+z_4\tau^2=0$. However,

Received January 12, 1981
The first author is partially supported by Australian Research Grant Commitee No. 7. L20, 205.