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A Residue Formula for Chern Classes Associated with Logarithmic Connections

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Introduction

There exist various residue theorems in complex analysis whose archetype is the residue theorem for meromorphic 1-forms on a compact Riemann surface. In this case a 1-form can be considered as a connection form of the trivial line bundle or of any holomorphic line bundle with flat representatives. Thus the fact that the sum of the residues of a meromorphic 1-form is zero means that the sum of the residues of a meromorphic connection of a holomorphic line bundle is equal to the Chern class (or number) of the bundle on a compact Riemann surface.

Now let us generalize the situation to higher dimensional cases. Let E be a holomorphic vector bundle over a compact complex manifold M, and let D be a meromorphic connection of E. Then we hope that there may exist some relations between the residues of D and the Chern classes of E. In this paper it is shown that if D is logarithmic and if the pole Z of D satisfies certain conditions, then there are relations among the residues of D, the pole Z and the Chern classes of E. Our theorem states:

If the pole Z of a logarithmic connection D is normally crossing, and if each irreducible component of Z is smooth and the intersection of any finite number of irreducible components of Z is connected, then the following relation holds in the cohomology group $H^k(M, \Omega^k)$

$$c_k(E)=(-1)^k \sum_{j_1,\cdots,j_k} c_k(\mathrm{Res}_{Z_{j_1}}D, \mathrm{Res}_{Z_{j_2}}D, \cdots, \mathrm{Res}_{Z_{j_k}}D) \prod_{t=1}^{k} c_1([Z_{j_t}]) ,$$

where $c_k(E)$ is the k-th Chern class of E, $\mathrm{Res}_{Z_j} D$ is the residue of D along the irreducible component Z_j of Z, $c_k(A_1, \cdots, A_k)$ is the completely polarized form of the k-th Chern polynomial $c_k(A)$, $c_1([W])$ is the Chern

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