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A Characterization of Homogeneous Self-dual Cones

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Introduction

A *convex cone* V in the n-dimensional real number space $\boldsymbol{R}^n$ is a non-empty open subset of $\boldsymbol{R}^n$ satisfying the following conditions:

(1) If $x \in V$ and $\lambda \in \boldsymbol{R}$ with $\lambda > 0$, then $\lambda x \in V$.

(2) If $x, y \in V$, then $x + y \in V$.

(3) V contains no full straight line.

We denote by $G(V)$ the group of all linear automorphisms of V, that is,

$$G(V) = \{A \in GL(n);\ AV = V\} .$$

If the group $G(V)$ acts transitively on V, then V is called *homogeneous*. Let $\langle\ ,\ \rangle$ be an inner product in $\boldsymbol{R}^n$. Then the *dual cone* V^* of V with respect to the inner product $\langle\ ,\ \rangle$ is defined by

$$V^* = \{y \in \boldsymbol{R}^n;\ \langle x, y\rangle > 0 \text{ for every } x \in \bar{V} - (0)\} ,$$

where $\bar{V}$ is the topological closure of V in $\boldsymbol{R}^n$. A cone V is called *self-dual* if the dual cone V^* of V with respect to a suitable inner product coincides with V. The *characteristic function* φ_V of V is defined on V by

$$\varphi_V(x) = \int_{V^*} \exp - \langle x, y\rangle dy ,$$

where dy is a canonical Euclidean measure on $\boldsymbol{R}^n$. The characteristic function of a homogeneous convex cone V is determined uniquely up to a constant factor by the following property:

$$\varphi_V(Ax) = \varphi_V(x)/|\det A|$$

for every $x \in V$, $A \in G(V)$. Let us take a system of linear coordinates $(x_1, x_2, \cdots, x_n)$ of $\boldsymbol{R}^n$. Then using the characteristic function we can define a $G(V)$-invariant Riemannian metric g_V on V by

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