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Some Examples of Analytic Functionals and their Transformations

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Introduction

Recently the theory of analytic functionals with non-compact carrier has been studied by many mathematicians. For example, see Morimoto [4], [5], Morimoto-Yoshino [6], [7], de Roever [8], Sargos-Morimoto [9], Zharinov [10]. However, it seems only a few examples have been given in the literature. In this paper, we will construct several examples of analytic functionals which are closely related to special functions. Furthermore we calculate the Fourier-Borel transformation and the Avanissian-Gay transformation of the analytic functionals. Some of them were already employed in Morimoto-Yoshino [7]. We will show that, in many cases, the Avanissian-Gay transformation $G_T(w)$ is a generating function of orthogonal polynomials and that the integral representation of the Fourier-Borel transformation by the Avanissian-Gay transformation is a classical formula of special functions. We confine ourselves to one dimensional case.

§1. Fundamental space $Q(L; k')$ and its dual space $Q'(L; k')$.

Let L be a convex compact set or a set of the following form: $L=[a, \infty)+i[-k, k]$, $i=\sqrt{-1}$, $k'\in \boldsymbol{R}$, $a\in \boldsymbol{R}$ and $k\geqq 0$. We denote by $Q_b(L_\epsilon; k'+\epsilon')$ the space of all functions $f(\zeta)$ holomorphic in the interior of L_ϵ which satisfy the following estimate:

$$\sup\{|f(\zeta)|\exp((k'+\epsilon')\xi);\ \zeta=\xi+i\eta\in L_\epsilon\}<+\infty\ ,$$

where $L_\epsilon=[a-\epsilon, \infty)+i[-k-\epsilon, k+\epsilon]$. Taking the inductive limit following the restriction mappings as $\epsilon>0$ and $\epsilon'>0$ tending to zero, we define the fundamental space $Q(L; k')$ as follows:

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