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Random Iteration of Unimodal Linear Transformations

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Introduction

Starting from the model of May, $f_\lambda(x)=\lambda x(1-x)$, $(0\leqq\lambda\leqq 4, x\in[0, 1])$, many works have been done on the topological and measure theoretical study of one-parameter family of one-dimensional transformations [1]~[8]. Especially, these works treat the phenomena of bifurcation, that is, the change of the behaviour of orbits according to the change of the parameter. In the case of the model of May, it is considered that the parameter λ expresses the characteristics of the species considered and that the value x expresses the population; and also the population of the $(n+1)$-st generation x_{n+1} is determined by the population of the n-th generation x_n by $x_{n+1}=f_\lambda(x_n)$.

In this paper we are concerned with a random family $\{f_\alpha; a\leqq\alpha\leqq b\}$ of transformations of an interval I into itself. On one hand, the random family of transformations may serve as more realistic models, e.g., models of population dynamics, if one takes into account of the randomness of the environment. On the other hand, there may appear some interesting situations. For example, the random system $\{f_\alpha\}$ may be mixing (a fortiori, exact ([10])), although each transformation f_α is not mixing.

We formulate the problem in the following manner. Let $\{f_\alpha; a\leqq\alpha\leqq b\}$ be a one-parameter family of transformations of an interval I into itself, and let $X_1, X_2, \cdots, X_n, \cdots$ be a sequence of independent and identically distributed random variables defined on a probability space (Ω, P) with $a\leqq X_n\leqq b$. Then, for each $\omega\in\Omega$, $x_0\in I$ is transformed to $x_1=f_{X_1(\omega)}(x_0)$, $x_2=f_{X_2(\omega)}(x_1), \cdots, x_n=f_{X_n(\omega)}(x_{n-1}), \cdots$. Our aim is to investigate the behaviour of the orbit $\{x_n; n\geqq 0\}$ for almost all $\omega\in\Omega$.

In this paper we only treat the following simplest case where the one-parameter family of transformations is that of unimodal linear transformations, that is,

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