

Ambiguous Numbers over $P(\zeta_3)$ of Absolutely Abelian Extensions of Degree 6

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Let K be an abelian number field of degree 6 over the rational number field P and suppose K contains a primitive 3rd root ζ_3 of unity. Then the ambiguous number of $K/P(\zeta_3)$ is 3^{2t-2} when 3 unramifies in $K/P(\zeta_3)$ and it is 3^{2t-1} when 3 ramifies in $K/P(\zeta_3)$ where $t+1$ is the number of prime numbers which ramify in K/P .

Let Γ be the genus field of K/P , then Γ/K is unramified and the number of these ideal classes of K which are principal in Γ is a multiple of $(\Gamma:K)$ and it is larger than $(\Gamma:K)$ if $t \geq 2$.

§1. Preliminaries.

Throughout this paper we shall use the following notations.

P The rational number field.

ζ_n A primitive n -th root of unity.

In this paper, the conductor of K is the minimal number f such that $K \subset P(\zeta_f)$ when K is abelian over P .

I_K The group of ideals in K .

P_K The group of principal ideals in K .

$h_K = [I_K: P_K]$ The class number of K .

$\mathfrak{A} \sim 1$ An ideal \mathfrak{A} is principal in the field.

$\mathfrak{A} \sim \mathfrak{B}$ Ideals \mathfrak{A} and \mathfrak{B} are contained in a same ideal class in the field.

We call $\mathfrak{A} \in I_K$ an ambiguous ideal if $\mathfrak{A}^\sigma = \mathfrak{A}$ for all $\sigma \in \text{Gal}(K/k)$ and we call $\mathfrak{A} \in I_K$ an ambiguous class ideal if $\mathfrak{A}^{1-\sigma} \in P_K$ for all $\sigma \in \text{Gal}(K/k)$.

$A_{0,K/k}$ The subgroup of I_K/P_K consisting of classes each of which contains an ambiguous ideal for K/k .

$a_{0,K/k}$ The order of $A_{0,K/k}$.

$A_{K/k}$ The subgroup of I_K/P_K consisting of classes each of which