On Modules over a Serial Ring Whose Endomorphism Rings are Quasi-Frobenius

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Introduction

The purpose of this paper is to establish several necessary and sufficient conditions for a module over a serial ring to have a quasi-Frobenius endomorphism ring.

In the study of properties of modules, it is greatly important to investigate their endomorphism rings. By Schur's Lemma the endomorphism ring of a simple module is a division ring, and we have enough knowledge about the endomorphism rings of modules over a semi-simple ring. Here we shall investigate the following problem:

PROBLEM. *Find a necessary and sufficient condition for a module U over a ring R to have a quasi-Frobenius endomorphism ring.*

Quasi-Frobenius rings are one of the most important classes of rings which are not semi-simple; in fact, a group algebra KG of a finite group G over a field K such that char $(K)||G|$ is not semi-simple, but it is quasi-Frobenius. As for the problem in the case U being a faithful module over a quasi-Frobenius ring, C. W. Curtis [1] gave a sufficient condition and K. Morita [6] obtained a necessary and sufficient condition. Recently J. A. Green [4] and H. Sawada [11] showed that a certain nonfaithful module over a group algebra of a finite group with a split (B, N)-pair has a Frobenius endomorphism algebra. Stimulated with Sawada's result [10], Green [4] gave a necessary condition for our problem in the case of U being a module over a group algebra under a certain assumption, and again Sawada [12] extended Green's result. On the other hand, K. Morita gave a sufficient condition for the above problem in the case U being a module over an Artinian ring (cf. Remark 14). However, each of these conditions is not a necessary