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On Homogeneous Convex Cones of Non-Positive Curvature

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Introduction

It is well known that homogeneous convex cones play an important role in the theory of homogeneous bounded domains (cf. e.g., [3], [7]). On a homogeneous convex cone V, a canonical Riemannian metric is defined which is closely related to the Bergman metric of the tube domain over V. With respect to this Riemannian metric, every homogeneous self-dual cone is a Riemannian symmetric space of non-positive sectional curvature (cf. [2], [8]). But it is little known about the Riemannian geometric properties of homogeneous non-self-dual cones. The main purposes of the present paper are to give a necessary condition for a homogeneous convex cone to be of non-positive sectional curvature with respect to the canonical metric and to determine such cones of rank 3 or of low dimensions.

The relation between homogeneous convex cones and homogeneous affine hyperspheres has been studied by Calabi [1] and Sasaki [9]. In § 1, we will recall some of definitions and the fundamental results on homogeneous convex cones and homogeneous affine hyperspheres from [13], [1] and [9]. In § 2, by using results of Sasaki [9] and Meschiari [5], we will see that every homogeneous convex cone with dimension $\geqq 2$ is homothetically equivalent to a product Riemannian manifold of a homogeneous hyperbolic affine hypersphere and the half line of positive real numbers (Proposition 2.1). As an application of this and a result in [10] or [12], a characterization for a homogeneous hyperbolic affine hypersphere to be Riemannian symmetric with respect to the affine metric will be given (Theorem 2.2). By making use of a result in Calabi [1] we will see that the Ricci curvature of a homogeneous convex cone is always non-positive (Theorem 2.3). In § 3, by using the results obtained in [12] we will calculate explicitly the curvature tensor of the canonical metric (Lemmas 3.1 and 3.2) and give a sufficient condition

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