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Construction of Aspherical Manifolds from Special G-Manifolds

Hiroshi NAKAMURA

Gakushuin University
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Introduction

Let G be a compact connected Lie group acting smoothly and effectively on a manifold X. We say that X is a (smooth) special G-manifold (see K. Jänich [6]) if for each $x \in X$ the slice representation $G_x \to GL(V_x)$ is the direct sum of a transitive and a trivial representation. In this case the orbit space $M=X/G$ is a differentiable manifold with boundary. K. Jänich showed that a special G-manifold X is constructed by a Lie group G, an orbit space M and an admissible orbit fine structure over M (roughly speaking, isotropy groups of G at $x \in X$).

Note that the following fact is known: If G is abelian, then $S[U_A] \cong \prod [G] \cong [M; BG]$ (see [6, Corollary 1]). That is, the isomorphic class $[X]$ depends only on the isomorphic class of the G-principal bundle P, and the class $[X]$ corresponds to a homotopy class of maps of M into the classifying space BG. But actually the homotopy groups of X can not be computed directly even if the homotopy groups of M are computable. In general also we do not know whether this X is an *aspherical* (i.e., its universal covering is contractible) manifold or not.

In this paper we give a condition that the special G-manifold is aspherical. In this case it is known from the result of Conner and Raymond [1, Theorem 5.6] that G is a toral group and all isotropy groups are finite. And under this condition it follows from Lemma 1 that the orbit structure U_A over M is a family of U_α which is isomorphic to $\boldsymbol{Z}_2$. And our main result is the following

THEOREM 1. *Let T^k be a k-dimensional toral group ($k>0$), M^m an m-dimensional compact connected differentiable manifold with boundary $\partial M=\bigcup_{\alpha\in A} B_\alpha$, where B_α is a connected component ($m>0$). Let* $(\boldsymbol{Z}_2)_A =$

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