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Moduli Space of Polarized del Pezzo Surfaces and its Compactification

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Introduction

A non-singular rational projective surface X over an algebraically closed field k is called a del Pezzo surface if the inverse of the canonical sheaf ω_X^{-1} is ample. It is called a del Pezzo surface of degree d if $\omega_X^{-1}\cdot\omega_X^{-1}=d$. It is known that the degree of a del Pezzo surface is at most 9 and the surface is isomorphic to $\boldsymbol{P}^2$ if $d=9$, $\boldsymbol{P}^1\times\boldsymbol{P}^1$ or $\boldsymbol{F}_1$ if $d=8$, the image of $\boldsymbol{P}^2$ under a monoidal transformation with center $(9-d)$-closed points in general position (cf. Definition 1) if $1\leqq d\leqq 7$ ([2]). Let X be a del Pezzo surface of degree $d\leqq 7$ and $f\colon X\to\boldsymbol{P}^2$ be a monoidal transformation of $\boldsymbol{P}^2$ with center $(9-d)$-points in general position. We call the sheaf $f^*\mathcal{O}_{P^2}(1)$ a contraction sheaf on X.

In this article we first construct the moduli space of del Pezzo surfaces of degree d $(1\leqq d\leqq 7)$ together with a contraction sheaf, and next construct its compactification in the sense of Definition 7.

In §1, we realize the moduli space of del Pezzo surfaces of degree $d(1\leqq d\leqq 7)$ as the geometric quotient by $PGL(2)$ of the open subspace U_d of $\mathrm{Sym}^{9-d}\boldsymbol{P}^2$, where U_d consists of the points which represent $(9-d)$-points in general position in $\boldsymbol{P}^2$.

In §2, to construct a "good" compactification of our moduli space, we take a blowing up of the subspace containing U_d with the center outside of U_d and next take its universal categorical quotient. Then we see that a point on the boundary corresponds to an irreducible reduced surface with possible A_1-singularities, which is isomorphic to the image of monoidal transformation of $\boldsymbol{P}^2$ with the center $(9-d)$-points allowing at most double points, where double point means a subscheme defined by an maximal primary ideal $\mathscr{I}$ in $\mathcal{O}_{P^2}$ such that $\dim_k\mathcal{O}_{P^2}/\mathscr{I}=2$ as a k-vector space.

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