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Configurations and Invariant Gauss-Manin Connections of Integrals I

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Introduction

A sequence of m polynomials $f_1, \cdots, f_m$ in $C[x_1, \cdots, x_n]$ defines a configuration of hyper-surfaces $S_j: f_j=0$ in C^n. The space of such configurations, being parametrized by the coefficients of polynomials, can be regarded as an analytic space. On this space, the integrals

$$\text{(J)} \qquad \int \exp[f_0] f_1^{\lambda_1} \cdots f_m^{\lambda_m} dx_1 \wedge \cdots \wedge dx_n$$

$$\text{(J}'\text{)} \qquad \int f_0^{\lambda_0} f_1^{\lambda_1} \cdots f_m^{\lambda_m} dx_1 \wedge \cdots \wedge dx_n$$

satisfy Gauss-Manin connections or equivalently holonomic systems in the sense of S. S. K. (See [8], [16] and [18].) But generally it seems difficult to get their explicit formulae in global forms. According to the method which has been developed in [1] and [5], in this note we shall give *the Gauss-Manin connections* for the above integrals *in invariant expressions* with respect to certain algebraic groups which act on them in a natural way, *when f_0 is quadratic and $f_1, \cdots, f_m$ are all linear* (see the formulae EI_0, EII_0-EII_p, $EIII_0$, EIV_0 and EV_p). These equations generalize *the Schläfli formula* for the volume of a spherical simplex (see [2]) and *Appell's hyper-geometric functions of type* (F_4) (see [10]). In case where f_0 is linear, they have been computed in terms of logarithmic forms and simple rational 1-forms of Grassmann coordinates attached to the configuration of hyper-planes (see [3]).

In Part II of this note, we shall show from the results obtained in Part I, that in case where the exponents $\lambda_0, \lambda_1, \cdots, \lambda_m$ are all integers, the integrals can be expressed by means of logarithmic connections of basic algebraic invariants, so that they become *hyper-logarithms* in the sense

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