TOKYO J. MATH.
VOL. 6, NO. 1, 1983

Exposed Points in Function Algebras

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(Communicated by J. Wada)

In this paper we consider some properties of exposed points in the unit ball of function algebras. In § 1 we give some characterizations of exposed points in the unit ball U of certain function algebras. Also we consider conditions so that U can be equal to the closed convex hull of exposed points of U. In § 2, some examples are given.

Introduction

Let X be a compact Hausdorff space and A a *function algebra* on X, i.e., a uniformly closed subalgebra of $C(X)$ that contains the constants and separates points of X, where $C(X)$ denotes the Banach algebra of complex-valued continuous functions on X with the supremum norm. By U we denote the unit ball of A, i.e., $U=\{f \in A: \|f\| \leqq 1\}$. We recall the notion of exposed points of U. A function f in U is called an *exposed point* of U if there exists L in A^* such that $L(f)=1=\|L\|$ and $\operatorname{Re} L(g)<1$ for $g \in U$, $g \neq f$, where $\operatorname{Re} L(g)$ is the real part of $L(g)$. It is clear that every exposed point is an extreme point but the converse is not always true.

Characterizations of exposed points have been investigated in [1], [3], [7], [8], [9] and so on. Especially, Phelps [7] gave some interesting results on logmodular algebras. Moreover Fisher [3] and Serizawa [8] gave extensions of the Phelps' results. In this paper we give some generalizations of Phelps' and Fisher's results.

We here assume the following condition.

(*) *There exist (pairwise disjoint) closed sets X_i in X $(i=1, 2, \cdots)$ such that $A|_{X_i}$ is closed in $C(X_i)$ and $\bigcup_{i=1}^{\infty} X_i$ is dense in X.*

For each i, we denote by A_i the restriction of A to X_i. M_A and M_{A_i} will denote the maximal ideal space of A and A_i, respectively. Then

Received November 30, 1981