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A Kummer Congruence for the Hurwitz-Herglotz Function

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Introduction

There has been much interest in the arithmetic properties of singular values of modular functions. Among the various topics studied are properties these singular values share with the Bernoulli numbers. Recently, Katayama [6] established analogues of the von Staudt-Clausen theorem for singular values of the Hurwitz-Herglotz function (defined below). In this paper, it will be shown that the singular values of the Hurwitz-Herglotz function also satisfy a Kummer congruence. Kummer congruences for singular values of Eisenstein series and for Eisenstein series of higher level have been demonstrated by H. Lang [8], [9].

§1. Kummer congruences.

Let k and r be integers with $k>r\geqq 1$ and let p be prime with $p-1\nmid k$. The classical Kummer congruence [7] states that the Bernoulli numbers satisfy

$$\sum_{s=0}^{r}(-1)^{s}\binom{r}{s}\frac{B_{k+s(p-1)}}{k+s(p-1)}\equiv 0(\mathrm{mod}\ p^{r})\ . \tag{1.1}$$

In the study of p-adic L-series, Iwasawa introduced generalized Bernoulli numbers B_{χ_f}, associated to characters χ_f of conductor f. Carlitz [3] proved a Kummer congruence for these generalized Bernoulli numbers. He showed that with k, p, and r as before and with $p\nmid f$, then

$$\sum_{s=0}^{r}(-1)^{s}\binom{r}{s}\frac{B_{\chi_f}^{k+s(p-1)}}{k+s(p-1)}\equiv 0(\mathrm{mod}\ p^{r})\ . \tag{1.2}$$

A related Kummer congruence was established by Vandiver [11] who

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