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Remarks on Stability for Semiproper Exceptional Leaves

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Introduction

A leaf of a codimension one foliation of a closed manifold is called *stable* if it has a saturated tubular neighborhood foliated as a product. About 1950, G. Reeb [12] (See also A. Haefliger [6].) showed that a compact leaf is stable if and only if it has a trivial holonomy group. It seems reasonable to conjecture that a proper leaf with a finitely generated fundamental group will be stable if it has a trivial holonomy group. (Note that the fundamental groups of compact leaves are always finitely generated and see also T. Inaba [11].) In fact, in 1976, T. Inaba [9], [10] extended Reeb's original theorem for proper leaves with finitely generated fundamental groups of codimension one foliations of closed three-manifolds. But this result is false if the fundamental groups of the leaves are not finitely generated. (See H. Imanishi [8].) In this paper, we extend Inaba's result for semiproper leaves and show that this extension is also false for leaves with infinitely generated fundamental groups by constructing a counterexample explicitly.

Section 1 gives basic definitions and fundamental properties of holonomy. Section 2 shows that Inaba's result is valid for semiproper leaves as well. Section 3 summarizes the result of G. Hector [7] for use in Section 4. Section 4 is devoted to constructing an example of unstable semiproper exceptional leaves without holonomy.

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§1. Introduction to the techniques.

First of all we recall some definitions and basic notions. Throughout this paper, $\mathscr{F}$ will denote a transversely orientable $C^r(0\leqq r\leqq\infty)$ codimension one foliation with C^∞ leaves of a closed manifold M and $\mathscr{L}$ will

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