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Classification of Periodic Maps on Compact Surfaces: I

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Introduction.

A homeomorphism $f: M \to M$ of a space M onto itself is called a periodic map on M with period n if $f^n = \text{identity}$ and $f^k \neq \text{identity}$ $(1 \leqq k < n)$. We say that a periodic map f on M is *equivalent* to a periodic map f' on M' if there exists a homeomorphism $h: M \to M'$ such that $fh = hf'$. In this paper, we will obtain classification of orientation-preserving periodic maps on compact orientable surfaces. Classification of orientation-reversing periodic maps on compact orientable surfaces and periodic maps on compact non-orientable surfaces will be given in the forthcoming paper [5].

We will consider a pair (f, M) where M is a compact connected surface and f is a periodic map on M with period n. Let $\mathscr{S}_k = \mathscr{S}_k(f) = \{x \in M;\ f^k(x) = x,\ f^i(x) \neq x\ (1 \leqq i < k)\}$ and $\mathscr{S} = \mathscr{S}(f) = \bigcup_{k=1}^{n-1} \mathscr{S}_k(f) = \{x \in M;\ 1 \leqq \exists\, k < n,\ f^k(x) = x\}$, say *a singular set* of f. Let P_n be a set of (f, M) satisfying the condition that $\mathscr{S}(f)$ consists of finite points in $\dot{M}$ (may be empty). For an element (f, M), we obtain its orbit space M/f from M by the identification of x with $f(x)$ for $x \in M$.

PROPOSITION 1 (Whyburn [4]). *The orbit space M/f is a compact surface.*

Let $p: M \to M/f$ be a canonical map. Then p is an n-fold cyclic branched covering map of M/f with a branched set $p(\mathscr{S}(f))$. For a compact connected surface X and a set S of finite points in $\dot{X}$, we denote by $P_n(X, S)$ a set of elements (f, M) of P_n satisfying the following conditions;

(1) the orbit space M/f is homeomorphic to X,

(2) the canonical map $p: M \to X$ is an n-fold cyclic branched covering map with a branched set S.

Suppose that (f, M) is equivalent to (f', M'). Clearly there exists a

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