

The Factorization of H^p and the Commutators

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Introduction

In [2] and [8], R. Coifman, R. Rochberg, G. Weiss and A. Uchiyama obtained the factorization theorems for $H^p(\mathbb{R}^n)$ by the singular integral operators. Recently S. Chanillo [1] obtained the factorization theorems for $H^1(\mathbb{R}^n)$ by the fractional integral operators. In this paper we think about the factorization theorems for $H^p(\mathbb{R}^n)$, $p < 1$, by the fractional integral operators and we apply the results to the boundedness of certain commutator operators.

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§1. The definition and the results.

DEFINITION. We say that K is a Calderón-Zygmund kernel if

$$K \not\equiv 0, \quad \int_{S^{n-1}} K(x') dx' = 0,$$

where dx' is the element of "area" of the sphere $|x|=1$,

$$K(rx) = r^{-n} K(x) \quad \text{when } r > 0 \text{ and } x \neq 0, \\ |K(x) - K(y)| \leq |x - y| \quad \text{when } |x| = |y| = 1,$$

and define

$$Kf(x) = P. V. \int_{\mathbb{R}^n} K(x-y) f(y) dy, \\ K'f(x) = P. V. \int_{\mathbb{R}^n} K(y-x) f(y) dy.$$