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A Note on Hasse's Theorem Concerning the Class Number Formula of Real Quadratic Fields

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Let p be a prime with $p\equiv 1 \pmod 4$ and h the class number of the real quadratic field $Q(\sqrt{p})$. Let $\varepsilon>1$ be a fundamental unit of $Q(\sqrt{p})$. As well-known, the Dirichlet's class number formula is stated in the form

$$\varepsilon^h=\frac{\prod_b \sin\frac{\pi b}{p}}{\prod_a \sin\frac{\pi a}{p}}, \tag{1}$$

where a and b runs over quadratic residues and quadratic non-residues between 0 and $p/2$ respectively. As h is a positive integer, the right-hand side of (1) is a unit in $Q(\sqrt{p})$. So ε^h is written in the form $u+v\sqrt{p}$, $u, v\in \mathbf{Q}$. The explicit formula of u and v is given by H. Hasse. (See [1].) In this paper we shall prove an alternative form of Hasse's theorem, which is slightly simpler in structure.

Let g be a fixed positive quadratic non-residue mod p and let $\boldsymbol{a}=(a_1, \cdots, a_n)$ and $\boldsymbol{b}=(b_1, \cdots, b_n)$ be systems of $n=(p-1)/4$ quadratic residues a_ν and quadratic non-residues b_ν with $0<a_\nu$, $b_\nu<p/2$. Furthermore let $\boldsymbol{x}=(x_1, \cdots, x_n)$, where $-(g-1)\leqq x_\nu\leqq g-1$ and any x_ν is odd or even, according as g is even or odd, be a solution of the congruence, respectively,

$$\boldsymbol{ax}=a_1x_1+\cdots+a_nx_n\equiv a_\nu \pmod p\,,$$
$$\boldsymbol{ax}=a_1x_1+\cdots+a_nx_n\equiv b_\nu \pmod p\,,$$

and

$$\boldsymbol{ax}=a_1x_1+\cdots+a_nx_n\equiv 0 \pmod p\,.$$

We write

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