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Functions Which Operate by Composition on the Real Part of a Banach Function Algebra

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Introduction

In this paper we study functions which operate on a Banach space which is the real part of a Banach function algebra. We say that A is a *Banach function algebra* if A is a Banach algebra lying in $C(X)$, the algebra of all complex-valued continuous functions on a compact Hausdorff space X which separates the points of X and contains constant functions. The history of this problem probably begins with J. Wermer's paper [8] in which he proved that the real part of a non-trivial function algebra is not closed under multiplication. $\mathrm{Re}\,A=\{u\in C_R(X): \exists f\in A,\ \mathrm{Re}\,f=u\}$, the space of the real part of a Banach function algebra A with the norm $N(\cdot)$ on a compact Hausdorff space X, is complete with the norm $N_R(\cdot)$ defined as follows. For each u in $\mathrm{Re}\,A$

$$N_R(u)=\inf\{N(f): f\in A,\ \mathrm{Re}\,f=u\}\ .$$

Suppose that h is a real-valued continuous function on a non-degenerate interval I, we say that h *operates by composition on* $\mathrm{Re}\,A$ if $h\circ u$ is in $\mathrm{Re}\,A$ whenever $u\in\mathrm{Re}\,A$ has range in I. J. Wermer's theorem is made a change in the wording that $t\mapsto t^2$ does not operate by composition on the real part of a non-trivial function algebra. Obviously each affine function has such a property for any Banach function algebra. It is natural to consider whether this result may be extended to any Banach function algebra. However, we easily find many counter examples for which the question does not hold, e.g.,

$$C^{(n)}[0,1],\ A(\Gamma)=\left\{f\in C(\Gamma): \sum_{-\infty}^{\infty}|\hat{f}(n)|<\infty\right\}\ ,$$

where Γ is the unit circle in the complex plane and $\hat{f}(n)$ is the n-th Fourier coefficient, and so on.

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