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A Note on Rings with Finite Local Cohomology

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Introduction

Let A be a Noetherian local ring of $\dim A=n$ and $\mathfrak{m}$ the maximal ideal of A. Let $H_{\mathfrak{m}}^i(\cdot)$ stand for the i^{th} local cohomology functor relative to $\mathfrak{m}$. Then we say that A has *finite local cohomology*, if the A-module $H_{\mathfrak{m}}^i(A)$ is finitely generated for every $i\neq n$.*) In this note we shall characterize rings with finite local cohomology in terms of d-sequences. Recall that a sequence $x_1, x_2, \cdots, x_r$ of elements in A is called a *d-sequence* if the equality

$$(x_1, \cdots, x_{i-1}): x_j=(x_1, \cdots, x_{i-1}): x_i x_j$$

holds whenever $1\leqq i\leqq j\leqq r$ ([5]). With this definition our result is stated as follows:

THEOREM. *The following conditions are equivalent.*

(1) *A has finite local cohomology.*

(2) *There exists an integer $N>0$ such that every system of parameters of A contained in $\mathfrak{m}^N$ is a d-sequence.*

When this is the case, $\mathfrak{m}^N\cdot H_{\mathfrak{m}}^i(A)=(0)$ for all $i\neq n$.

Our theorem is a natural extension of Huneke's characterization of Buchsbaum rings. Recall that a Noetherian local ring A is called *Buchsbaum* if the difference

$$l_A(A/\mathfrak{q})-e_{\mathfrak{q}}(A)$$

is an invariant of A not depending on the choice of a parameter ideal $\mathfrak{q}$ of A, where $l_A(A/\mathfrak{q})$ and $e_{\mathfrak{q}}(A)$ denote the length of the A-module $A/\mathfrak{q}$ and the multiplicity of A relative to $\mathfrak{q}$, respectively ([10]). Buchsbaum rings have, as is well-known (cf. [6]), finite local cohomology, and

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*) In [8], rings with finite local cohomology are called *generalized Cohen-Macaulay*.