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On the Genus Field in Algebraic Number Fields

Mitsuko HORIE

Tokyo Metropolitan University

Introduction

In 1951, Hasse [7] started from the principal genus theorem of Gauss and gave a class field-theoretic interpretation of the genus theory of quadratic number fields. Subsequently, Leopoldt [11] extended the theory to abelian fields, and Fröhlich [1] generalized it to arbitrary number fields. In his paper [11], Leopoldt studied the "Auflösung" of groups of numerical characters and gave an ideal-theoretic characterization of genus fields and a genus number formula of abelian fields. By origin, the principal genus in the Gauss' classical theory is defined as the kernel of numerical characters which are called the genus characters. Hasse [9] asserted that Leopoldt's characterization of genus fields of abelian fields can restated as the following: the principal genus of an abelian field is the kernel of norm residue symbols. And recently, Gold [3] showed that the principal genus in the wide sense of a relative Galois extension is also characterized as the kernel of norm residue symbols. On the other hand, Furuta [2], using idele, obtained a genus number formula in the wide sense of relative Galois extensions. The genera that the above authors except Furuta and Gold treated in their papers were in the narrow sense.

The genus field, the genus number and the principal genus are defined as follows.

DEFINITION. Let K/k be an arbitrary extension of finite algebraic number fields. The narrow (wide) genus field K^* of K/k is the maximal extension of K, which satisfies the following conditions:

(i) K^* is a composite of K and an abelian extension of k,

(ii) no finite (and no infinite) prime in K ramifies in K^*/K.

The genus number of K/k is the degree $[K^*: K]$. Clearly, the genus field K^* is a class field over K, and we call the ideal group of K corresponding to K^* the principal genus.

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