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# Some Properties of Vector Bundles on the Flag Variety $Fl(r, s; n)$

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## Introduction

In this article we would like to begin to study vector bundles over flag varieties. It is well-known that every vector bundle over a projective line $P_1$ is a direct sum of line bundles. Thus for a given vector bundle over a variety $F$, after restricting it to each line contained in $F$, by examining which line bundles appear as direct summands, we can analyze its properties. This method was developed in Van de Ven [12], Barth [1], and Hartshorne [6], when $F$ is a projective space.

In § 1 in this article, we construct explicitly a component $\hat{X}$ of the Hilbert scheme parametrizing "straight lines" in the flag variety $F=Fl(r, s; n)=\{(P_r, P_s)\,|\,P_r\subset P_s\subset P_n\}$ $(0\leqq r\leqq s<n)$. (In our case the union $Q$ of two copies of $P_1$ intersecting transversally at one point appears over a divisor $\hat{\Delta}$ in $\hat{X}$ as a deformation of the usual straight line. Thus it is necessary that we call $Q$ "a line" as well. However, vector bundles over $Q$ have simple properties and it is not an obstacle for our purposes.)

In § 2 we give a theorem concerned with Schubert varieties in $F$ and $\hat{X}$. Now according to § 1, we can define tautological homogeneous vector bundles not only on $F$ but also on $\hat{X}$. We will show in Theorem 2.1 that there is a systematic correspondence between Chern classes of tautological vector bundles on $F$ and those on $\hat{X}$.

Let $\hat{\Gamma}$ be the universal family of subvarieties of $F$ over $\hat{X}$. We have a diagram

$$\begin{array}{ccc} & \hat{\Gamma} & \\ {}^{\hat{\alpha}}\swarrow & & \searrow^{\hat{\beta}} \\ \hat{X} & & F \end{array}$$

such that for every point $x\in\hat{X}$, $\hat{\beta}\hat{\alpha}^{-1}(x)$ is the subvariety corresponding to $x\in\hat{X}$. Let $\mathscr{M}$ be a vector bundle on $F$ with $c_1(\mathscr{M})=0$ such that

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