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On Homeomorphisms with Pseudo-Orbit Tracing Property

Nobuo AOKI

Tokyo Metropolitan University

Let $f: X \circlearrowleft$ be a homeomorphism of a compact metric space onto itself and Ω be the non-wandering set of f. The following is a main result of this paper.

THEOREM 1. *If (X, f) has the pseudo-orbit tracing property, then so does (Ω, f).*

This is a problem proposed by A. Morimoto [5].

Let d be a metric function of X. A sequence of points $\{x_i\}_{i\in(a,b)}$ $(-\infty\leqq a<b\leqq\infty)$ is called a *δ-pseudo-orbit* (abbrev. p.o.) of f if $d(f(x_i), x_{i+1})<\delta$ for $i\in(a, b-1)$. A sequence $\{x_i\}_{i\in(a,b)}$ is called to be *ε-traced* by $x\in X$ if $d(f^i(x), x_i)<\varepsilon$ holds for $i\in(a, b)$. We say that (X, f) has the *pseudo-orbit tracing property* (abbrev. P.O.T.P.) if for every $\varepsilon>0$ there is $\delta>0$ such that every δ-p.o. of f can be ε-traced by some point $x\in X$. Given $x, y\in X$ and $\alpha>0$, x is *α-related* to y (written $x\overset{\alpha}{\sim}y$) if there are α-pseudo-orbits of f such that $x_0=x, x_1, \cdots, x_k=y$ and $y_0=y, y_1, \cdots, y_l=x$. If $x\overset{\alpha}{\sim}y$ for every $\alpha>0$, then x is *related* to y (written $x\sim y$). The *chain recurrent set* of f, R is $\{x\in X: x\sim x\}$.

Recall that $\Omega=\{x\in X$: for every neighborhood U of x, $f^n(U)\cap U\neq\emptyset$ for some $n\geqq1\}$. Clearly $\Omega\subset R$ and both sets are f-invariant and closed (a set E will be called f-invariant when $f(E)=E$). Assume that (X, f) has the P.O.T.P., then $\Omega=R$. For, if $x\in R$ then for every $\alpha>0$ there is $\alpha'>0$ with property of the P.O.T.P.; i.e., for every α'-p.o. $\{x_i\}$ such that $x_0=x, x_1, \cdots, x_k=x$, there is $y\in X$ with $d(f^i(y), x_i)<\alpha$ $(0\leqq i\leqq k)$. Hence $U_\alpha(x)\cap f^{-k}(U_\alpha(x))\neq\emptyset$ where $U_\alpha(x)=\{y\in X: d(x, y)<\alpha\}$, and so $x\in\Omega$.

We proceed with a sequence of lemmas leading to the proof of Theorem 1. For the following (L. 1), (L. 2) and (L. 3), it is assumed that (X, f) has the P.O.T.P.. Denote by per(f) the set of all periodic points of f.

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