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Classification of T^2-bundles over T^2

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By T^2, we mean a two dimensional torus $\mathbf{R}^2/\mathbf{Z}^2$. The purpose of this paper is to classify fiber bundles which have T^2 as fibers and base spaces. (Simply we call them *T^2-bundles over T^2*.)

In view of bundle isomorphisms, we obtain Theorem 4 which gives necessary and sufficient condition that two bundles are bundle isomorphic. By this theorem, one might determine even by computer whether two bundles are isomorphic or not.

In view of homeomorphism types of total spaces, we obtain Theorem 5 which says that total spaces are homeomorphic if and only if their fundamental groups are isomorphic.

In Theorem 3, we show that any T^2-bundle over T^2 is isomorphic to one of some standard types of bundles.

§ 1. Notations and definitions.

Given $A, B \in GL(2, \mathbf{Z})$ such that $AB=BA$, and $m, n \in \mathbf{Z}$, we construct a T^2-bundle over T^2 denoted by $\pi: M(A, B; m, n) \to S$, as follows.

Denote by $\begin{bmatrix} x \\ y \end{bmatrix}$ the point of $T^2 = \mathbf{R}^2/\mathbf{Z}^2$ corresponding to $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2$. Let $F=T^2$, $S=T^2$, and we define

$$M(A, B; 0, 0) = F \times \mathbf{R}^2/\sim$$

where

$$\left(\begin{bmatrix} s \\ t \end{bmatrix}, \begin{pmatrix} x+1 \\ y \end{pmatrix}\right) \sim \left(\begin{bmatrix} A\begin{pmatrix} s \\ t \end{pmatrix} \end{bmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}\right),$$

and

$$\left(\begin{bmatrix} s \\ t \end{bmatrix}, \begin{pmatrix} x \\ y+1 \end{pmatrix}\right) \sim \left(\begin{bmatrix} B\begin{pmatrix} s \\ t \end{pmatrix} \end{bmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}\right).$$

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