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Maximal Surfaces in the 3-Dimensional Minkowski Space L^3

Osamu KOBAYASHI

Keio University

A surface in the 3-dimensional Minkowski space $L^3=(\boldsymbol{R}^3, dx^2+dy^2-dz^2)$ is called a *space-like* surface if the induced metric on the surface is a positive definite Riemannian metric. A space-like surface with vanishing mean curvature is called a *maximal surface*.

In this paper, we give the Weierstrass-Enneper representation formulas for maximal surfaces (§ 1), and exhibit various examples (§ 2). In particular, we determine the maximal surfaces which are rotation surfaces or ruled surfaces (§§ 3, 4). In contrast with the case of minimal surfaces in the Euclidean 3-space, where a rotation minimal surface (resp. a ruled minimal surface) is locally congruent to a catenoid (resp. a helicoid), there are various types of maximal rotation or maximal ruled surfaces. For example, maximal Enneper's surfaces appear as rotation or ruled surfaces. In the last section, we give some explicit examples which show that the so-called Bernstein property does not hold, in general, without an ellipticity condition (§ 5).

§ 1. Weierstrass-Enneper formulas for maximal surfaces in L^3.

For a space-like surface in L^3, the Gauss map is defined to be a mapping which assigns to each point of the surface the unit normal vector at the point. Therefore, its image can be regarded as contained in a space-like surface $H^2=\{(x, y, z)\in L^3;\ x^2+y^2-z^2=-1\}$, which has constant negative curvature -1 with respect to the induced metric. We define a stereographic mapping σ for H^2 in the following way

$$\begin{aligned}(1.1)\qquad &\sigma\colon \boldsymbol{C}\backslash\{|\zeta|=1\}\longrightarrow H^2\,; \qquad \zeta\longmapsto\left(\frac{-2\operatorname{Re}\zeta}{|\zeta|^2-1},\ \frac{-2\operatorname{Im}\zeta}{|\zeta|^2-1},\ \frac{|\zeta|^2+1}{|\zeta|^2-1}\right)\\ &\text{and}\quad \sigma(\infty)=(0, 0, 1)\,.\end{aligned}$$

That is, $\sigma(\zeta)$ is the intersection of H^2 and the line joining $(\operatorname{Re}\zeta, \operatorname{Im}\zeta, 0)$

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