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A Construction of an Invariant Stable Foliation by the Shadowing Lemma

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Introduction

There are many studies on the dynamical properties of one-dimensional maps. For instance, asymptotic behavior and the existence of invariant measures were studied in [1], [2] and [3]. In contrast, in the case of two-dimensional maps the results obtained are not so many. So, it would be usuful to investigate whether there exist two-dimensional maps which can be reduced to one-dimensional maps.

In this paper, to clarify how the behavior of not necessarily differentiable two-dimensional maps is related to that of one-dimensional maps, we investigate the existence of an invariant stable foliation of two-dimensional maps by using the shadowing lemma.

Let $I=[0, 1]$ and f be a map of piecewise C^{2}-class from I into itself; i.e., there is a finite sequence $0=c_0<c_1<\cdots<c_N=1$ of points in I such that if $I_i=[c_i, c_{i+1})$ then the restriction of f to I_i is C^2 and there exist $\lim_{x\to c_{i+1}-0}(d^n/dx^n)f(x)$ $(n=0, 1, 2)$. A sequence of points $\{x_n\}_{n\geq 0}$ is called an *ε-pseudo-orbit* of f iff $|f(x_n)-x_{n+1}|<\varepsilon$ for $n\geqq 0$. Denote sometimes by I_x the interval I_i that contains a point x. A sequence $\{x_n\}_{n\geq 0}$ is called *β-traced* by $\xi\in I$ iff $|f^n(\xi)-x_n|<\beta$ and $f^n(\xi)\in I_{x_n}$ for $n\geqq 0$. We say that (I, f) has the *pseudo-orbit-tracing property* (abbrev. P.O.T.P) iff for every $\beta>0$ there exists $\varepsilon=\varepsilon(\beta)>0$ such that every $\varepsilon(\beta)$-pseudo-orbit is β-traced by some point $\xi\in I$. We claim that our definition of P. O. T. P is not the same as in R. Bowen (p. 74, (4)).

Throughout this paper, we denote by $f'(x)$ the right or left differential coefficient ($f'_+(x)$ or $f'_-(x)$, respectively) at a discontinuity point x if there is no confusion.

For convenience we write

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