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Homeomorphisms with the Pseudo Orbit Tracing Property of the Cantor Set

Masahito DATEYAMA

Tokyo Metropolitan University
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Let X be a compact metric space with metric d, and f be a homeomorphism from X onto itself. A sequence $\{x_i\}_{i=-\infty}^{\infty}$ is said to be a *δ-pseudo-orbit* of f if $d(fx_i, x_{i+1})<\delta$ holds for all $i\in \boldsymbol{Z}$. (X, f) is said to have the *pseudo orbit tracing property* (abbrev. P.O.T.P.) if for every $\varepsilon>0$ there is $\delta>0$ such that, for every δ-pseudo-orbit $\{x_i\}_{i=-\infty}^{\infty}\subset X$, there exists an $x\in X$ such that $d(f^i x, x_i)<\varepsilon$ for all $i\in \boldsymbol{Z}$. Let $C\subset[0, 1]$ be the Cantor set: i.e. C is the set of the numbers $x\in[0, 1]$ with $x=3^{-1}a_1+3^{-2}a_2+\cdots$ ($a_i=0$ or 2 for $i\geqq 1$). We denote by $\mathscr{H}(C)$ the set of all homeomorphisms on C, and by $\mathscr{P}(C)$ the set of all homeomorphisms with the P.O.T.P.. Define the metric $\bar{d}$ on $\mathscr{H}(C)$ by $\bar{d}(f, g)=\max_{x\in C} d(fx, gx)$, $f, g\in\mathscr{H}(C)$. Then $\mathscr{H}(C)$ is a Banach space.

In this paper we prove:

THEOREM. *$\mathscr{P}(C)$ is dense in $\mathscr{H}(C)$.*

For $r\geqq 1$, we call the set $C\cap[3^{-r}i, 3^{-r}(i+1)]$ $(0\leqq i\leqq 3^r-1)$ a *Cantor subinterval with rank* r if $C\cap(3^{-r}i, 3^{-r}(i+1))\neq\varnothing$. We denote by $I(i, r)$, the i-th Cantor subinterval with the rank r from the left. Clearly $C=\bigcup_{i=1}^{2^r} I(i, r)$ and $I(i, r)=I(2i-1, r+1)\cup I(2i, r+1)$. We call $g\in\mathscr{H}(C)$ a *generalized permutation* if there exists $r\geqq 1$ such that the following i) and ii) hold:

i) For every $1\leqq i\leqq 2^r$, there exist $s=s(i)\geqq 1$ and $1\leqq j=j(i)\leqq 2^s$ such that $g(I(i, r))=I(j, s)$, and

ii) For every $1\leqq i\leqq 2^r$, there exists $k=k(i)\in\boldsymbol{R}$ such that $g(x)=3^{r-s(i)}x+k$, $x\in I(i, r)$.

Denote by $\mathscr{G}$ the set of all generalized permutations. Then $\mathscr{G}$ is dense in $\mathscr{H}(C)$. In fact, take $f\in\mathscr{H}(C)$ and $r\geqq 1$. Choose $s\geqq 1$ such that $d(x, y)<3^{-s}$ implies $d(fx, fy)<3^{-r}$. Then for every $1\leqq i\leqq 2^s$ there exists

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