Alexander Ideals of Graphs in the 3-Sphere

Dedicated to Professor Takizo Minagawa for his 70th birthday

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(Communicated by J. Wada)

Let \((P \subset S^n)\) be a pair of the oriented \(n\)-sphere \(S^n\) \((n \geq 3)\) and a finite subpolyhedron \(P\) of \(S^n\) with \(S^n - P\) connected. Using Fox's free differential calculus ([2], [3], [4]), S. Kinoshita [11] explained that for each non-negative integer \(d\) there is the \(d^{th}\) elementary ideal \(E_d\) of the fundamental group \(G(P) \equiv \pi_1(S^n - P)\), associated with each integral \((n-2)\)-cycle \(l\) on \(P\), so that the collection \(\{E_d\}\) forms a topological invariant of the position of \(P\) in \(S^n\). He also examined some fundamental properties of it in [11], [12] and [13].

In this paper we discuss the elementary ideals of finite 1-dimensional polyhedra in the 3-sphere \(S^3\) associated with the abelianizer, and give a necessary condition for the exterior of a connected 1-dimensional polyhedron to be retractible and boundary-retractible [9], (Theorems 3.1 and 3.2).

§ 1. Preliminaries.

Throughout the paper we work in the piecewise linear category.

By \(P\) we denote a finite 1-dimensional polyhedron with \(\mu\) components \(P_1, \ldots, P_\mu\), \(\mu \geq 1\). We denote by \(\beta_i\) the 1-dimensional Betti number of \(P_i\) for \(i = 1, \ldots, \mu\), and let \(\beta = \beta_1 + \cdots + \beta_\mu\). We always assume that \(\beta_i > 0\) for \(i = 1, \ldots, \mu\), and we will call such a pair \((P \subset S^3)\) of the 3-sphere \(S^3\) and its subpolyhedron \(P\) a graph in \(S^3\).

For a graph \((P \subset S^3)\), by the exterior \(M(P)\) of \(P\) we mean the closure of \(S^3 - N(P; S^3)\), where \(N(P; S^3)\) is a regular neighborhood of \(P\) in \(S^3\), and by \(G(P)\) we denote the fundamental group \(\pi_1(S^3 - P) \equiv \pi_1(M(P))\).

We shall consider finitely presentable groups and their finite presentations. For a finite presentation \(\langle x_1, \cdots, x_n | r_1, \cdots, r_m \rangle\) of a group \(G\),