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# A Remark on a Theorem of B. T. Batikyan and E. A. Gorin

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## Introduction

Let $X$ be a compact Hausdorff space and $\tilde{X}=\beta(N\times X)$ be the Stone-Čech compactification of $N\times X$, the direct product of the space of natural numbers $N$ and $X$. We consider a Banach space $E$ which satisfies that $E$ is a Banach space lying in $C(X)$ (resp. $C_R(X)$) with the norm $\|\cdot\|_E$ such that $\|u\|_\infty\leqq\|u\|_E$ for each $u$ in $E$ where $\|\cdot\|_\infty$ denotes the supremum norm and we also suppose that $E$ separates the points of $X$ and contains constant functions with $\|1\|_E=1$. Let $\tilde{E}=1^\infty(N, E)$ be the Banach space of all bounded sequences in $E$ with the norm $\|(f_n)\|_{\tilde{E}}=\sup_n\|f_n\|_E$. For every $(f_n)$ in $\tilde{E}$ we can suppose that $(f_n)$ is a bounded continuous function on $N\times X$ defined as $(f_n)(m, x)=f_m(x)$ for $(m, x)$ in $N\times X$. So we may suppose that $\tilde{E}$ is lying in $C(\tilde{X})$ (resp. $C_R(\tilde{X})$). We say that $E$ is *ultra-separating* on $X$ if $\tilde{E}$ separates the points of $\tilde{X}$ (cf. [2], [3], [4]).

### §1. A characterization for ultraseparability.

We say that $A$ is a Banach function algebra on $X$ if $A$ is a Banach algebra lying in $C(X)$ which separates the points of $X$ and contains constant functions. It is shown in B. T. Batikyan and E. A. Gorin [2] that ultraseparability for a Banach function algebra $A$ can be characterized as follows:

*There exist a natural number $m$ and $\delta>0$ such that for every pair of disjoint compact subsets $Y_1$ and $Y_2$ of $X$ there exist functions $f_1, f_2, \cdots, f_m$ and $g_1, g_2\cdots, g_m$ in the unit ball of $A$ which satisfy*

$$\sum_{i=1}^{m}(|f_i|-|g_i|)\geqq\delta \quad on \quad Y_1$$

$$\sum_{i=1}^{m}(|f_i|-|g_i|)\leqq-\delta \quad on \quad Y_2\,.$$

Let $\operatorname{Re}E=\{u\in C_R(X): \exists f\in E, \operatorname{Re}f=u\}$. Then $\operatorname{Re}E$ is also an above

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