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Weighted Norm Inequalities for Certain Pseudo-Differential Operators

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Introduction

Several authors (R. R. Coifman, C. Fefferman, R. A. Hunt, B. Mukenhoupt and R. L. Wheeden [1], [4], [6]) have shown that if T is the Hardy-Littlewood maximal operator or a classical singular integral operator, the weighted norm inequality

$$\int_{R^n}|Tf(x)|^p w(x)dx \leqq C\int_{R^n}|f(x)|^p w(x)dx$$

is valid if and only if the weight function w satisfies the A_p-condition (see (3) below). Recently, N. Miller [5] showed that the same thing is also true when T is the pseudo-differential operator of order 0. In this note, we shall show that even when T is the pseudo-differential operator whose symbol, $\sigma(x, \xi)$, satisfies the regularity condition on x weaker than in the symbol of the pseudo-differential operator of order 0, the same thing is also true.

A pseudo-differential operator σ with symbol $\sigma(x, \xi)$, defined initially on the Schwartz class $\mathscr{S}(\boldsymbol{R}^n)$, is given by

$$f \longrightarrow \sigma f(x) = \int_{R^n} \sigma(x, \xi)\hat{f}(\xi)e^{2\pi i x\cdot\xi}d\xi ,$$

where $\hat{f}$ denote the Fourier transform of f defined by

$$\hat{f}(\xi) = \int_{R^n} f(x)e^{-2\pi i x\cdot\xi}dx .$$

We shall say that the function $\sigma(x, \xi) \in C^\infty(\boldsymbol{R}^n \times \boldsymbol{R}^n)$ is a symbol of order m if it satisfies

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