Expansion of the Solutions of a Gauss-Manin System at a Point of Infinity

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Introduction

Let \( f(x) \) be a polynomial, in \( n \) complex variables \( x = (x_1, \ldots, x_n) \), with an isolated critical point and let \( F_0(t, x) \) be a deformation of \( f(x) \) with parameters \( t = (t_1, \ldots, t_m) \). Setting \( F = t_0 + F_0 \) with a distinguished parameter \( t_0 \), we shall investigate the differential system to be satisfied by the integral of type

\[
\frac{u}{\lambda} = \int \delta^{(2)}(F)dx \quad \text{or} \quad \int F^{-\lambda-1}dx \quad (dx = dx_1 \wedge \cdots \wedge dx_n),
\]

where \( \lambda \) is a (generic) complex number. Roughly speaking, such a Gauss-Manin system defines a meromorphic connection, on the space \( S \) of parameters \( (t_0, t) \), at most with poles along its discriminant variety \( D \). Thus, our attention will be paid to the many-valued holomorphic solutions on \( S \backslash D \) of the Gauss-Manin system. In “simple” examples, one can show that a fundamental system \( \Phi(t_0, t) \) of its many-valued holomorphic solutions can be expanded into a power series

\[
\Phi(t_0, t) = \sum_{r=0}^{\infty} \Phi_r(t_0) t_0^{-A-(r+1)I}
\]

convergent near the point \( (t_0, t) = (\infty, 0) \) at infinity, where \( -A \) is the matrix of exponents of \( f \) shifted by \( \lambda \). In the present article, we shall determine such an expansion of \( \Phi \) in an explicit manner for typical examples of Gauss-Manin systems.

Our computational results will be given in \( \S 3 \). The polynomial \( f(x) \) to be deformed is assumed there to belong to either of the types

\[
\begin{align*}
(1) & \quad f(x) = x_1^{p_1} + x_2^{p_2} + \cdots + x_n^{p_n} \\
(2) & \quad f(x) = x_1^{p_1} + x_1x_2^{p_2} + x_3^{p_3} + \cdots + x_n^{p_n}.
\end{align*}
\]

Received November 9, 1982