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An Environment of Quasi-Valuation Domains

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Introduction

Any domain W has an ordered group $G(W)$. This group, the set of non-zero principal fractional ideals of W with $xW \leqq yW$ if and only if xW contains yW, is called the group of divisibility of W. Let $K^{\times}=K\backslash\{0\}$ be the multiplicative group of quotient field of W and $U(W)$ the group of units of W, then $G(W)$ is order isomorphic to $K^{\times}/U(W)$, where $xU(W) \leqq yU(W)$ if and only if $y/x \in W$. It is wellknown that $G(W)$ is linearly ordered if and only if W is a valuation domain.

In section 1, to define a good preordered group (2.1), we study an additive abelian group admitting two co-linear preorder relations compatible with the group operation.

In section 2, using the basic results of section 1, we discuss some facts related to a domain W under the assumption that $G(W)$ is a good preordered group. Then W is dominated by a valuation domain V. We call this domain W a quasi-valuation domain; in particular, in case V is integral over W we call W a prevaluation domain. Furthermore, there are many similarities between quasi-valuation domains and valuation domains. In fact $V\backslash U(V)=W\backslash U(W)$. Then it is only natural that a quasi-valuation domain has some normalities. A quasi-valuation domain W is really seminormal, i.e., $\mathrm{Pic}(W)\rightarrow \mathrm{Pic}(W[X])$ is an isomorphism, where $\mathrm{Pic}(W)$ is the Picard group of W and X is an indeterminate. Therefore, for a domain R, it stands to reason that we should think about $\cap W_{\lambda}$, the intersection ranging over all quasivaluation domains containing R. This domain $R^{\sharp}=\cap W_{\lambda}$ is seminormal; $R^{\sharp}$ is not always the seminormalization R^{+} of R, however.

In section 3, we show that $R^{\sharp}$ is the largest subdomain R' of $\tilde{R}$ containing R such that, for all $p' \in \mathrm{Spec}(R')$, the canonical homomorphism $k(p' \cap R)\rightarrow k(p')$ is an isomorphism, where $\tilde{R}$ is the derived normal ring

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