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On Homeomorphisms with Markov Partitions

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Introduction

Let (X, d) be a compact metric space and f be a homeomorphism from X onto itself. f is called *expansive* if there exists $e>0$ such that $d(f^n(x), f^n(y))\leqq e$ for all $n\in \boldsymbol{Z}$ implies $x=y$. The number e is called an *expansive constant* of f. A sequence $\{x_i\}_{i\in \boldsymbol{Z}}$ of X is a *δ-pseudo orbit* of f if $d(f(x_i), x_{i+1})<\delta$ for all $i\in \boldsymbol{Z}$. We say that $x\in X$ *ε-traces* a sequence $\{x_i\}_{i\in \boldsymbol{Z}}$ of X if $d(f^i(x), x_i)<\varepsilon$ for all $i\in \boldsymbol{Z}$. f is called to have the *pseudo orbit tracing property* (abbrev. P. O. T. P.) if for any $\varepsilon>0$ there exists $\delta>0$ such that every δ-pseudo orbit of f is ε-traced by a point of X. These properties of f are independent of metrics for X compatible with original topology.

A typical example of expansive homeomorphisms with P. O. T. P. is given in [1, 2]; it is shown that expansive group automorphisms of solenoidal groups have P. O. T. P. though in general group automorphisms with P. O. T. P. are not expansive. Another example is obtained from an *expanding map* $g: X\rightarrow X$, that is, g is an onto open map and there are $\delta>0$ and $\lambda>1$ such that $d(x, y)<\delta$ implies $d(g(x), g(y))\geqq\lambda d(x, y)$. Such maps become homeomorphisms through inverse limit and it is known that these homeomorphisms are expansive and have P. O. T. P.. As these examples show, we can construct easily examples of expansive homeomorphisms with P. O. T. P. and they form a larger class than that of Anosov diffeomorphisms. It was posed as a problem in topological dynamics whether every expansive homeomorphism of a torus satisfying P. O. T. P. is topologically conjugate to a toral automorphism, and the technique in this paper is important to solve this problem ([6]).

Ja. G. Sinai constructed in [9] Markov partitions for Anosov diffeomorphisms of compact C^∞ manifolds. After that R. Bowen [3, 5] constructed the same partitions for basic sets of Axiom A diffeomorphisms

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