TOKYO J. MATH.
VOL. 8, NO. 1, 1985

# A Complex Continued Fraction Transformation and Its Ergodic Properties

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## Introduction

In this paper we introduce a continued fraction algorithm $T$ of complex numbers and investigate metrical properties of this algorithm. $T$ is defined on the domain $X=\{z=x\alpha+y\bar{\alpha};\ -(1/2)\leqq x,\ y\leqq(1/2)\}$ $(\alpha=1+i)$ by $Tz=(1/z)-[1/z]_1$, where $[z]_1$ denotes $[x+(1/2)]\alpha+[y+(1/2)]\bar{\alpha}$ for a complex number $z=x\alpha+y\bar{\alpha}$. This map $T$ induces a continued fraction expansion of $z\in X$,

$$z=\frac{1|}{|a_1}+\frac{1|}{|a_2}+\frac{1|}{|a_3}+\cdots$$

where each $a_i$ is of the form $n\alpha+m\bar{\alpha}$ for some integers $n$ and $m$. We give fundamental definitions and properties of this continued fraction algorithm $T$ in §1.

To investigate approximation properties of continued fractions, the dual continued fraction

$$\frac{1|}{|a_n}+\frac{1|}{|a_{n-1}}+\cdots+\frac{1|}{|a_2}+\frac{1|}{|a_1}$$

plays an important role. In §2, we define the algorithm $S$ which induces $T$-dual continued fraction. By using this algorithm $S$, we show that

$$\left|z-\frac{p_n}{q_n}\right|\leqq\frac{\sqrt{2}}{|q_n|}$$

for each $z\in X$ and $n\geqq1$, where $p_n/q_n$ denotes the $n$-th approximant introduced by $T$, and we also show that the value $\sqrt{2}$ is the best possible constant.

In §3 we construct the natural extension map $R$ of $T$ by combining

Received September 30, 1983
Revised May 24, 1984