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# Representation Theory of Weyl Group of Type $C_n$

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## Introduction

The irreducible representations of the Weyl group of type $C_n$, which we denote by $W(C_n)$, is well known. They are constructed for example using the fact that $W(C_n)$ is the semi direct product of the symmetric group $\mathfrak{S}_n$ and an elementary abelian group of even order (cf. [1]). In this paper we take another approach. We use quite similar technique to construct irreducible representations to that of the symmetric group as in [2]. That is, our approach is to use two disjoint subgroups (horizontal and vertical) and their linear characters (Theorem 2). The irreducible characters are then also constructed explicitly using Schur function (Theorem 8). From this construction we deduce the analogue of Nakayama's formula for $W(C_n)$ to calculate the character value using Young diagram (Theorem 9). We also construct multiplicity formula in the induced representation of linear character of subgroup of type $W_B \cong \mathfrak{S}_{\lambda_1} \times \cdots \times \mathfrak{S}_{\lambda_r} \times W(C_{\mu_1}) \times \cdots \times W(C_{\mu_s})$ (Theorem 7). As an application we determine the $I$-set of $W$-graph corresponding to the irreducible representation of $W(C_n)$ (Theorem 11). This application seems to show some significance of our method.

## §1. The construction of irreducible representations.

**1.1** Let $W=W(C_n)$ be the Weyl group of type $C_n$. To begin with, we realize $W$ as a subgroup of $\mathfrak{S}_\Omega$, the symmetric group on the set $\Omega$ where $\Omega=\{1, 2, \cdots, n, -n, \cdots, -2, -1\}$ as follows:

$$W=\{x \in \mathfrak{S}_\Omega | xw_0=w_0x\}$$

where $w_0=(1, -1)(2, -2)\cdots(n, -n) \in \mathfrak{S}_\Omega$. For $x$ in $W$, we decompose $x$ into cyclic permutations as an element of $\mathfrak{S}_\Omega$. Then we get two kinds

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