TOKYO J. MATH.
VOL. 8, No. 1, 1985

Schur Indices of Some Finite Chevalley Groups of Rank 2, I

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Introduction

Let $\boldsymbol{F}_q$ be a finite field with q elements, of characteristic p. Let us consider the special orthogonal group $SO_5(q)$ of degree 5 over $\boldsymbol{F}_q$, the conformal symplectic group $CSp_4(q)$ of degree 4 over $\boldsymbol{F}_q$ and the Chevalley group $G_2(q)$ of type (G_2) over $\boldsymbol{F}_q$. If $p=2$, then $CSp_4(q)\simeq\boldsymbol{F}_q^*\times Sp_4(q)$, and the irreducible characters of $Sp_4(2^f)$ were described by H. Enomoto [24]. The character table of $CSp_4(q)$, q odd, was obtained by K. Shinoda in [19] (according to him, the table had also been obtained by S. Reid independently). The characters of $G_2(q)$ were calculated by B. Chang and R. Ree [4] when $p\neq 2, 3$ and by Enomoto [7, 8] when $p=2, 3$ ([8] has not been published yet). The complete table of characters of $SO_5(q)$, q odd, seems to have not been obtained yet. However much information about it can be gotten from G. Lusztig's theory [15] on the classification of the irreducible representations of finite classical groups (see §3 below). As to the rationality-properties of the characters of these groups, R. Gow has proved in [10] that all the irreducible characters of $Sp_4(q)$, q even, have the Schur index 1 over the field $\boldsymbol{Q}$ of rational numbers. Therefore, if $p=2$, all the irreducible characters of $CSp_4(q)$ $(\simeq\boldsymbol{F}_q^*\times Sp_4(q))$ and $SO_5(q)$ $(\simeq Sp_4(q))$ have the Schur index 1 over $\boldsymbol{Q}$. In this paper we shall prove the following.

MAIN THEOREM. *Suppose q is odd. Then all the irreducible characters of $SO_5(q)$, $CSp_4(q)$ and $G_2(q)$ have the Schur index 1 over $\boldsymbol{Q}$.*

It can be shown that all the irreducible characters of $G_2(2^f)$ also have the Schur index 1 over $\boldsymbol{Q}$. This case will be treated in the subsequent paper.

Now let G be a simple adjoint algebraic group defined and split over $\boldsymbol{F}_q$, and $G(q)$ be the group of its $\boldsymbol{F}_q$-points. If the rank of G is 1, then $G(q)=PGL_2(q)$, and if the rank is 2, then $G(q)$ is a homomorphic image

Received October 28, 1983