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# Paracomplex Structures and Affine Symmetric Spaces

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(Dedicated to the memory of Prof. Mikao Moriya)

## Introduction

Let $M$ be a smooth manifold. A splitting of the tangent bundle $T(M)$ of $M$ into the Whitney sum of two subbundles $T^{\pm}(M)$ is called an *almost paracomplex structure* on $M$, if $T^{\pm}(M)$ have the same fiber dimension. This is characterized by a (1, 1)-tensor field $I$ satisfying the conditions: $I^2=\mathrm{id}$, and $\pm 1$-eigenspaces of $I_p$ $(p\in M)$ are the fibers of $T^{\pm}(M)$ over $p$. If the distributions on $M$ defined by $T^{\pm}(M)$ are both completely integrable, then the almost paracomplex structure is called a *paracomplex structure*. These two structures were originally introduced by P. Libermann in 1952 ([5], [6]), in analogy with almost complex or complex structures. Libermann also introduced, although in somewhat vague fashion, the notions of *parahermitian metrics* and *parakähler metrics*, which are the paracomplex analogues of hermitian and Kähler metrics. It should be noted that a parakähler manifold has naturally a symplectic structure. The main interest is thus to what extent one can develop the theory of paracomplex manifolds in parallel with the theory of complex manifolds.

In this article, we introduce a class of affine symmetric spaces, called *parahermitian symmetric spaces*, a paracomplex analogue of hermitian symmetric spaces. §1 is devoted to some definitions and basic properties on paracomplex structures. In §2 we give the definition of parahermitian symmetric spaces and include Lie algebraic considerations. In §3 we give a group-theoretic characterization for an affine symmetric coset space $G/H$ with $G$ semisimple to be parahermitian symmetric (cf. Theorems 3.6 and 3.7). In §4 we consider a relation between parahermitian symmetric spaces of semisimple Lie groups and symmetric $R$-spaces (cf. Proposition 4.1 and Theorem 4.3). Finally we give the infinitesimal classification of parahermitian symmetric spaces with semisimple automorphism groups,

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