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A Note on Characteristic Functions and Projectively Invariant Metrics on a Bounded Convex Domain

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Introduction

The purpose of this note is to propose two metrics on a bounded convex domain, which are projectively invariant and seem to have similar nature to the Blaschke metric.

To recall the Blaschke metric let us take a bounded convex domain Ω in $\boldsymbol{R}^n$ and consider the differential equation

$$(\#) \qquad \det \partial^2 u/\partial x^i \partial x^j = (-u)^{-n-2} \quad \text{on} \quad \Omega\,, \qquad u=0 \quad \text{on} \quad \partial\Omega\,.$$

Since this equation has the unique negative strictly convex solution as is shown in [13] for dimension 2 and [5] in general, we can define a metric $-(1/u)d^2u$. This metric was first considered by Blaschke [1] and Tzitzèica [16] and can be thought of a possible generalization of the Hilbert metric of the ball. We call this shortly the *Blaschke metric* of the domain Ω. This is known to be complete; [4], [14].

A bounded pseudoconvex domain in $\boldsymbol{C}^n$ on the other hand has in general several biholomorphically invariant metrics. We would like to take two of them. One is the Einstein-Kähler metric, which exist at least under some smoothness condition on the boundary [6], and the other is the Bergman metric. Let us pay our attention to the special case where the domain is a tube over a cone V. In this case let Ω be a nontrivial hyperplane section of the cone. Then these metrics have special forms by the invariance and must have their correspondences on Ω. Namely, on the one hand in Appendix A, we show the Einstein-Kähler metric on the tube domain corresponds to the Blaschke metric on Ω.

On the other hand in §1, corresponding to the Bergman kernel, we will define a kernel function on Ω. In the course of this we need to

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