TOKYO J. MATH.
VOL. 8, NO. 1, 1985

On Regular Fréchet-Lie Groups VIII

Primordial Operators and Fourier Integral Operators

Yoshiaki MAEDA, Hideki OMORI, Osamu KOBAYASHI
and Akira YOSHIOKA

*Keio University, Science University of Tokyo, Keio University
and Tokyo Metropolitan University*

In this paper, we prove that the group of invertible Fourier-integral operators of order 0 is a regular Fréchet-Lie group with the Lie algebra $\sqrt{-1}\mathscr{P}^1$, where $\mathscr{P}^1$ is the totality of pseudo-differential operators of order one with the real principal symbols. As stated in the preface of [8], this is the main purpose of this series. So, this paper is the final one of our series.

§1. Preliminaries and the statement of main theorem.

1.1. Notations.

Throughout this paper, we use mainly the same notations as in [8]. Let N be a closed C^∞ riemannian manifold and TN and T^*N be the tangent bundle and the cotangent bundle of N respectively. A point of TN (resp. T^*N) is denoted by $(x; X)$ (resp. $(x; \xi)$). Denote by $\mathring{T}^*N$ the complement of the zero section in T^*N, i.e., $T^*N-\{0\}$ in the notation of [8]. A symplectic diffeomorphism φ of T^*N is called to be *positively homogeneous* of degree one, if it commutes with multiplication by positive scalars. That is, if we write φ as $\varphi(x; \xi)=(\varphi_1(x; \xi); \varphi_2(x; \xi))$, then it satisfies $\varphi_1(x; r\xi)=\varphi_1(x; \xi)$, $\varphi_2(x; r\xi)=r\varphi_2(x; \xi)$, for any $r>0$.

Let $\mathscr{D}_\Omega^{(1)}$ be the totality of symplectic diffeomorphisms of $\mathring{T}^*N$ of positively homogeneous of degree one. Then, we have proved that *$\mathscr{D}_\Omega^{(1)}$ is naturally identified with $\mathscr{D}_\omega(S^*N)$, the group of all contact transformations on the unit sphere bundle S^*N, and $\mathscr{D}_\Omega^{(1)}$ is a regular Fréchet-Lie group* (cf. [6] and Theorem 6.4 in [11]).

Now, in this paper, all derivatives of functions, tensors, etc., on TN, T^*N and S^*N, etc. are taken by using a normal coordinate system at the considered point (cf. [8], §1, and [9], §1, (15)).

Received May 11, 1983