TOKYO J. MATH.
VOL. 8, NO. 2, 1985

Local Topological Properties of Differentiable Mappings II

[Dedicated to Professor Morio Obata on his sixtieth Birthday]

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(Communicated by K. Kojima)

Introduction

In the preceding paper [2], it was shown that *almost every* C^∞ map-germ: $(R^n, 0)\to(R^p, 0)$, $n\leqq p$, has rather good topological structures. In particular it was shown that they are topologically equivalent to the cones of topologically stable mappings of S^{n-1} into S^{p-1}, where the cone of a mapping $f\colon X\to Y$ is the mapping $Cf\colon X\times[0, 1)/X\times\{0\}\to Y\times[0, 1)/Y\times\{0\}$ defined by $Cf(x, t)=(f(x), t)$. Here *almost every* is used in the rather strong sense that the complement of the set of these map-germs should have infinite codimension in the space of all C^∞ map-germs.

This paper has two purposes. One is to show similar generic properties for the remaining case $n>p$. The other is to show, as an application of these generic properties, that for almost every mapping into the plane $f\colon(R^n, 0)\to(R^2, 0)$ a Poincare-Hopf type equality, in some cases the Morse inequalities as well, holds between the Betti numbers of the set $f^{-1}(0)\cap S_\varepsilon^{n-1}$ and the indices of the singular points of f appearing around the origin, where $S_\varepsilon^{n-1}=\{x\in R^n \mid \|x\|=\varepsilon\}$ and ε is supposed to be small. The index of a singular point of a mapping into the plane will be defined later in this section.

Let us explain these properties more precisely. $J^r(n, p)$ is the set of the r-jets of all C^∞ map-germs: $(R^n, 0)\to(R^p, 0)$. For a positive number $\varepsilon>0$, we set

$$D_\varepsilon^m=\{x\in R^m \mid \|x\|\leqq\varepsilon\}\ ,$$
$$S_\varepsilon^{m-1}=\{x\in R^m \mid \|x\|=\varepsilon\}\ .$$

THEOREM 1. *For each positive integer r, there exists a closed*

Received August 23, 1984
This work was partially supported by Australian Research Grant Committee No. 7, L20.205.