TOKYO J. MATH.
VOL. 8, NO. 2, 1985

Extended Alexander Matrices of 3-Manifolds II

—Applications—

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§1. Statement of results.

In this paper we study 3-manifolds obtained from S^3 by Dehn surgery along knots. Let p be a positive integer and q be an integer relatively prime to p.

DEFINITION. For a knot $k \subset S^3$, let $L(p, q; k)$ be a 3-manifolds obtained from S^3 by Dehn surgery along k with coefficient p/q.

Note that when k is an unknot $L(p, q; k)$ means just a lens space $L(p, q)$. Clearly $L(p, q; k)$ is a homology lens space and $H_1(L(p, q; k)) = \boldsymbol{Z}_p$ is generated by an element corresponding to a meridian of the tubular neighbourhood $N(k)$ of k. We denote this element by t. Then an element of a group ring $\boldsymbol{Z}H_1(L(p, q; k)) = \boldsymbol{Z}[\boldsymbol{Z}_p]$ can be represented by a polynomial of t with integer coefficients where $t^p = 1$. We give a necessary condition for $L(p, q; k)$ to be a lens space.

THEOREM 1. *Let k be a knot with the Alexander polynomial Δ_k. Suppose that $L(p, q; k)$ is homeomorphic to $L(p, q')$. Let r, r' be integers such that $rq \equiv 1(p)$ and $r'q' \equiv 1(p)$. Then there are $u \in \boldsymbol{Z}[\boldsymbol{Z}_p]$ and $l, s \in \boldsymbol{Z}$ such that $(p, s) = 1$ which satisfy the equation*

$$(1 + t + \cdots + t^{r-1})\Delta_k(t) \equiv \pm t^l u\bar{u}(1 + t^s + \cdots + t^{s(r'-1)}) \bmod (1 + t + \cdots + t^{p-1})$$

in $\boldsymbol{Z}[\boldsymbol{Z}_p]$.

As a corollary we can prove:

THEOREM 2. *Let k be a knot with trivial Alexander polynomial. Then $L(p, q; k)$ and $L(p, q')$ can be homeomorphic only if $q \equiv \pm q'(p)$ or $qq' \equiv \pm 1(p)$.*

Received May 28, 1984
Revised September 27, 1984