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On the Isotropy Subgroup of the Automorphism Group of a Parahermitian Symmetric Space

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Introduction

Let (M, I, g) be a parahermitian symmetric space [2] which is identified with a co-adjoint orbit of a real simple Lie group with Lie algebra $\mathfrak{g}$. Such a manifold M can be expressed as an affine symmetric coset space $G/C(Z)$, where G is the analytic subgroup generated by $\mathfrak{g}$ in the simply connected Lie group corresponding to the complexification of $\mathfrak{g}$, and $C(Z)$ is the centralizer in G of an element $Z\in\mathfrak{g}$ satisfying the condition (C) (see §1).

The purpose of this paper is to study the isotropy subgroup $C(Z)$—the number of its connected components and the structure of the identity component. Our method here is classification-free. A main result is Theorem 3.7, which is efficiently used in Kaneyuki and Williams [3], [4], in applying the method of geometric quantization.

NOTATIONS.

G^0 the identity component of a Lie group G,
G_α the set of elements in G left fixed by an automorphism α of G,
$\boldsymbol{C}^*$ (resp. $\boldsymbol{R}^*$) the multiplicative group of non-zero complex (resp. real) numbers,
$\boldsymbol{R}^+$ the multiplicative group of positive real numbers,
i $=\sqrt{-1}$.

§1. Symmetric triples.

Let $\mathfrak{g}$ be a real simple Lie algebra and $\mathfrak{h}$ be a subalgebra of $\mathfrak{g}$ and σ be an involutive automorphism of $\mathfrak{g}$ such that $\mathfrak{h}$ is the set of σ-fixed elements in $\mathfrak{g}$. Then the triple $\{\mathfrak{g}, \mathfrak{h}, \sigma\}$ is called a simple symmetric triple. Suppose further that $\{\mathfrak{g}, \mathfrak{h}, \sigma\}$ satisfies the following condition (C) (which is equivalent to (C_3) in [2]):

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