

On Classification of Parahermitian Symmetric Spaces

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Introduction

The purpose of this paper is to give a method of classifying parahermitian symmetric spaces with semisimple automorphism groups up to local paraholomorphic equivalence (see Definition 1.1 for local paraholomorphic equivalence). The outline of the classification was given in our previous paper [1]. In §1 we reduce the problem to the classification of parahermitian symmetric coset spaces of semisimple Lie groups. Proposition 1.6 is the main result in §1. In §2 we consider the reduction of the problem to the case where the groups are simple. In §3 we give the main theorem (Theorem 3.4) which establishes a one-to-one correspondence between parahermitian symmetric spaces with simple automorphism groups and a certain class of simple graded Lie algebras which was worked out by Kobayashi-Nagano [2]. The explicit infinitesimal forms of these spaces are given in the previous paper [1].

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NOTATIONS. The Lie algebra of a Lie group G is denoted by the corresponding German small letter or $\text{Lie } G$. G^0 denotes the identity component of a Lie group G . $\phi_{*,p}$ denotes the differential of a map ϕ at p . $T_p(M)$ denotes the tangent space to M at p . id denotes the identity mapping. The Lie group homomorphism and the corresponding Lie algebra homomorphism are denoted by the same letter unless otherwise stated.

§ 1. Some properties of the automorphism groups.

Let (M, I) be an almost paracomplex manifold, and let \tilde{M} be a covering manifold of M and $\pi: \tilde{M} \rightarrow M$ be the natural projection.

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