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On Logarithmic Canonical Divisors on Threefolds

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Introduction

The aim of this paper is to give a numerical criterion for the logarithmic canonical or the logarithmic anti-canonical divisor on a threefold to be ample. As a corollary we obtain a practical definition of logarithmic Fano threefolds. Let V be a non-singular projective variety over an algebraically closed field of characteristic zero and $D=D_1+\cdots+D_S$ a reduced divisor whose components are smooth and crossing normally on V. We consider here such a pair (V, D), which is called a non-singular pair of dimension $n=\dim V$. Let K_V, or in short K, denote a canonical divisor on V. Then $K+D$ (resp. $-K-D$) is called the logarithmic canonical divisor (resp. logarithmic anti-canonical divisor) on V (cf. [3, Chap. 11]). We prove the following

THEOREM. *Let (V, D) be a non-singular pair of dimension* 3. *Then* (i) *under the condition that $\kappa(K+D, V)\geqq 0$, $K+D$ is ample if and only if $K+D$ is numerically positive; i.e. $(K+D)\cdot C>0$ for all curves C on V,* (ii) *under the condition that $\kappa(-K-D, V)\geqq 0$, $-K-D$ is ample if and only if $-K-D$ is numerically positive.*

COROLLARY (cf. [4]). *Let (V, D) be as in the Theorem. Then (V, D) is a logarithmic Fano threefold if and only if the following two conditions are satisfied.*

(a) *The linear system $|-K-D|$ is non-empty.*

(b) *$-K-D$ is numerically positive.*

PROOF. The if part follows from the Theorem.

Let (V, D) be a logarithmic Fano threefold. Applying Norimatsu Vanishing ([5, Theorem 1]) we deduce

$$H^i(V, \mathcal{O}_V(-K-D))=0 \quad \text{for} \quad i>0$$

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