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On Stable Ideals

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Introduction

Let A be a d-dimensional Cohen-Macaulay semi-local ring. We say A is equi-dimensional, if $\dim(A_M)=d$ for all maximal ideals M of A, or if A is a Macaulay ring of Nagata [3]. The length of an A-module E will be denoted by $\ell(E)$ or $\ell_A(E)$ to avoid ambiguity.

Sally proved in [5], [6], [7], and [8] that a d-dimensional Cohen-Macaulay local ring A with its maximal ideal M and multiplicity e, has the maximal embedding dimension $e+d-1$, if and only if the Hilbert-Samuel function $\ell(A/M^{n+1})$ of A equals a polynomial

$$P(n)=e\binom{n+d-1}{d}+\binom{n+d-1}{d-1}$$

for all $n\geqq 0$. In fact, more was proved in [8]: For A to have the maximal embedding dimension, it is sufficient that the above $P(n)$ is known to be the Hilbert-Samuel polynomial of A, or $\ell(A/M^{n+1})=P(n)$ for all large n. Our previous work [1] contains an extension of the first assertion: Let I be an open ideal of an equi-dimensional Cohen-Macaulay semi-local ring A of dimension d, then

$$\ell(I/I^2)=e+(d-1)\ell(A/I)\ ,$$

if and only if the Hilbert-Samuel function of I $\ell(A/I^{n+1})$ equals a polynomial

$$Q(n)=e\binom{n+d-1}{d}+\ell(A/I)\binom{n+d-1}{d-1}$$

for all $n\geqq 0$, where e is the multiplicity of I. In this paper, we shall show that the above conditions for I will be satisfied, if we know that

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