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On the Hilbert-Samuel Function

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Introduction

Let A be a Cohen-Macaulay semi-local ring of dimension d. The length of an A-module E will be denoted by $\ell(E)$. Let I be an open ideal of A which contains some power of the Jacobson radical of A, then the Hilbert-Samuel function $\ell(A/I^{n+1})$ of I equals

$$e_0\binom{n+d}{d}-e_1\binom{n+d-1}{d-1}+\cdots+(-1)^{d-1}e_{d-1}\binom{n+1}{1}+(-1)^d e_d$$

for large n. The coefficients e_k $(0\leqq k\leqq d)$ are called the normalised Hilbert-Samuel coefficients of I. e_k will be denoted by $e_k(I)$, if it is necessary to avoid confusion.

Assume that A is a Cohen-Macaulay local ring of dimension $d>0$, M the maximal ideal of A, and the residue field A/M infinite. Abhyankar [1] proved inequality

$$\ell(M/M^2)\leqq e+d-1$$

where $e=e_0(M)$. Sally [11] proved that equality holds if and only if there exists an ideal X generated by a system of parameters of A such that $M^2=XM$, and that if equality holds then

$$\ell(A/M^{n+1})=e\binom{n+d-1}{d}+\binom{n+d-1}{d-1}$$

for all $n\geqq 0$ ([12] Theorem 1). We know also

$$\ell(M/M^2)=e+d-1-\ell(M^2/XM)$$

for any ideal X generated by a system of parameters of A, such that $M^{n+1}=XM^n$ for large n ([13] Lemma 2.1). Furthermore, for a primary

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