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Homogeneity Theorems on Perfect Codes in Hamming Schemes and Generalized Hamming Schemes

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Introduction

Let F be a finite set of q elements, where $q>1$, q is not necessarily assumed to be a prime power, and let X be the set of all d-tuples over F. We may assume $F=\{0, 1, \cdots, q-1\}$ without loss of generality, and we regard X as an additive group. For $\boldsymbol{x}=(x_i)\in X$, $\boldsymbol{y}=(y_i)\in X$, we define the Hamming distance on X by $\partial(x, y)=|\{i\,|\,x_i\neq y_i\}|$, and distance relations R_i by $R_i=\{(\boldsymbol{x}, \boldsymbol{y})\in X\times X\,|\,\partial(\boldsymbol{x}, \boldsymbol{y})=i\}$ for $i=0, 1, \cdots, d$. Then $(X, \{R_i\}_{i=0}^d)$ is a symmetric association scheme, which is called a Hamming scheme, and is denoted by $H(d, q)$. A perfect e-error-correcting code in X (or a perfect e-code in $H(d, q)$) is a subset C of X such that for every $\boldsymbol{x}\in X$ there exists exactly one $\boldsymbol{c}\in C$ satisfying $\partial(\boldsymbol{x}, \boldsymbol{c})\leqq e$.

The classification of perfect e-codes in $H(d, q)$ is completed for the case $e\geqq 3$ by Tietäväinen, van Lint, Bannai, Reuvers, Best, Hong, and many others (see [4] for details). For the case $e=2$, the known perfect 2-codes have the following parameters (see [6, chapter V]):

(1) $d=1, 2$ (trivial codes)

(2) $d=5, q=2$ (binary repetition code)

(3) $d=11, q=3$ (ternary Golay code)

and they are unique up to isomorphism. Tietäväinen-van Lint [5, 10] showed that there exists no unknown perfect 2-code in $H(d, q)$, provided q is a prime power. But if q is not a prime power, the (non)existence problem remains open. We know two necessary conditions for the existence of a perfect e-code in $H(d, q)$ with q arbitrary.

The first is called the sphere packing condition. Let $S_e(\boldsymbol{c})$ denote the sphere of radius e with center $\boldsymbol{c}\in X$, i.e., $S_e(\boldsymbol{c})=\{\boldsymbol{x}\in X|\partial(\boldsymbol{x}, \boldsymbol{c})\leqq e\}$. Then a subset C of X is a perfect e-code in $H(d, q)$ if and only if $\{S_e(\boldsymbol{c})|\boldsymbol{c}\in C\}$ is a partition of X. Thus the following condition is necessary for the

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