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On the Decay of Correlation for Piecewise Monotonic Mappings I

Makoto MORI

The National Defence Academy
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Introduction

In this paper, we determine the decay rate of correlation for a certain class of piecewise linear mappings explicitly (for more general cases, we will mention in [11]) and apply it to the critical phenomena in dynamical system. The decay of correlation is already pointed out to be determined in terms of the Fredholm determinant on the physical level in [12]. However, since the Perron-Frobenius operator is generally not of trace class ([15]), it has not been proved except for Markov piecewise linear mappings from mathematical point of view. One of our aims is to give the proof to this assertion for the class of mappings F with constant slope λ which satisfies the conditions given below.

We will consider a power series Φ, called the Fredholm determinant (the reciprocal of the Artin-Mazur-Ruelle zeta function), associated with piecewise linear mapping F (whose definition will be given in §2) and the roots of $\Phi(1/\gamma)=0$ will be called Fredholm eigenvalues. By γ_1, γ_2, we denote the Fredholm eigenvalues which are the first and the second greatest in modulus (in fact, γ_1 equals the slope λ of the mapping F). Our main theorem is stated as follows:

THEOREM 0.1. i) *Suppose that $\gamma_1>\eta$. Then the following two statements are equivalent*:

1) *There exists an absolutely continuous invariant measure with which the dynamical system $((0, 1), \mu, F)$ is mixing.*

2) $$\rho(x)=-\lambda(\Phi'(1/\lambda))^{-1}\chi(1/\lambda; x)\geqq 0 ,$$

for any $x\in[0, 1]$, where the definition of $\chi(z; x)$ will be given in §2.

ii) *If the statements of* i) *hold, then the density function of μ equals*

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