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Hardy Spaces of 2-Parameter Brownian Martingales

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Introduction

In this note we shall study Hardy spaces and BMO spaces of martingales on the product Brownian spaces and their applications to function theory on the torus.

Hardy spaces and BMO spaces of martingales on the Brownian spaces were studied by N.Th.Varopoulos ([12], [13]) in connection with function theory on the unit circle $\boldsymbol{T}$. If we will deal with H^p and BMO martingales related to function theory on the torus $\boldsymbol{T}^2$, then we need to consider ones of 2-parameter on the product Brownian spaces (cf. [14]). J. Brossard and L. Chevalier built stochastic integral theory for such martingales and defined Hardy spaces H^p. They generalized the Burkholder-Davis-Gundy inequality to the product Brownian spaces ([4]). We here write K^p instead of H^p. H. Sato defined 2-parameter BMO martingales and proved $(K^1)^* = BMO$ ([10]).

After some preliminaries in §1, in §2 we define Hilbert transforms H_j on $K^p (j=1, 2, 3; 0<p<\infty)$ modeled after Hilbert transforms for 1-parameter martingales defined by N.Th.Varopoulos ([13]). In this section we prove equivalence of K^1-norm and $\|X\|_{L^1} + \sum_{j=1}^{3} \|H_j X\|_{L^1}$-norm (cf. Theorem 2.4). This extends a theorem of Varopoulos ([13, Theorem 3.2]). In §3 we study projections N and M introduced by N.Th. Varopoulos and obtain some results on them. Our main theorem is Theorem 3.5 which states that $\mathscr{H}^1(\boldsymbol{T}^2)$ (resp. $BMO(\boldsymbol{T}^2)$) is isomorphic to a closed complemented subspace of K^1(resp. BMO). This theorem implies several results as corollaries. Two of these are a theorem of Sato ([10]) and a theorem of Gundy-Stein ([7]). In §4 we concern with H^∞ of 2-parameter holomorphic martingales considered as abstract Hardy algebras and their applications. In this section we obtain several results on H^∞, which extend theorems of Varopoulos ([13]), for example, the density of H^∞ in H^p and that $\mathrm{Log}(X) \in BMOA$ for every

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